International Journal of Advances on Automotive and Technology
Promech Corp. Press, Istanbul, Turkey
Vol.1, No. 2 , pp. 89-99, April 2017

http://dx.doi.org.10.15659/ijaat.17.05.505
Manuscript Received October 20, 2016; Accepted March 10, 2017.

This paper was recommended for publication in revised form by Co-Editor Yasin Karagoz

MAIN PARAMETERS OF CAM PROFILE AND EFFECTS ON BEHAVIOR OF THE CAM PERFORMANCE

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Keywords: Main parameters, cam profile, acceleration jump, cam follower, valve spring
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ABSTRACT

In scope of this study, main parameters of cam profile are determined. Also, their effects on cam performance are examined starting with valve lift, velocity, acceleration curve. In order to be seen clearly cam behavior and obtain to needed smooth curve thanks to less jerk of cam follower, different valve lifts are calculated but more importantly one of the most critical design criteria on cam profile, acceleration jump as undesired influence, is investigated how to reduce & avoid. Primarily, relations related base circle & nose radius of cam profile, maximum lift of cam follower, lift angle of cam follower and acceleration jump are considered in terms of minimum acceleration jump, continuous contact, valve spring force & contact force-stress. Especially, determination of right valve spring stiffness is significant & affects other main parameters so it is plotted graphs of sudden acceleration change and described how it should be analyzed by analytical & using Matlab.

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INTRODUCTION

Valve train system for an engine is vital importance in terms of performance, fuel consumption, vibration and noise etc. Due to the fact that valve train system is main control mechanism and predetermined specified motion actor for engines, all effects on parts of systems should be considered during design process. Type of the valve train system is changeable acc.to engine placement & requirements but consequently, there are common parts for all types of system. These are the main ones: camshaft, cam follower, push rod, rocker arm, valve spring, valve etc.

Elements used in valve mechanism used to be under control motion of the engine. Basic motion of system is determined thanks to camshaft accordingly cam profile. A cam is rotating machine element which gives reciprocating or oscillating motion to another element known as follower. The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc. [1].

By means of computer aided design & analysis programs, experimental approach, optimization steps, it is possible to reach desired result for cam and valve train system design. If willing to get optimal and efficient valve system, first of all it should be started with cam profile determination because one of the first criteria coming from designer as output is valve lift. Designers require this dimension is as high as possible due to volumetric efficiency, but at the same time, require this dimension is as low as possible due to effected engine out size. Because of this mentioned conflict, first output of system should be determined carefully. According to valve lift dimension, base circle radius, nose radius will be calculated and plotted lift, velocity, acceleration graphs. We will be able to see how system parameters change according to all valve lift. From this initial point, it is possible to understand for per lift is necessary different nose, base circle radius in order to get optimal results.

Basic question for designer is what our optimization criteria are. In this paper, we will be able to recognize acceleration jump with graphs influences. Contrary to what is believed, maximum acceleration of follower is not merely the strongest criteria. It is not enough to take a decision on system behavior. It can be come across a situation which has high level of acceleration but acceleration jump is low. Although acceleration jump is low, due to high maximum acceleration designer should not directly eliminate this case. Because designer's first approach for cam profile optimization should be low acceleration jump.

In order to get low acceleration jump, change of base circle radius, nose radius, valve lift angle, maximum valve lift acc.to crank shaft angle will be obtained by using Matlab.

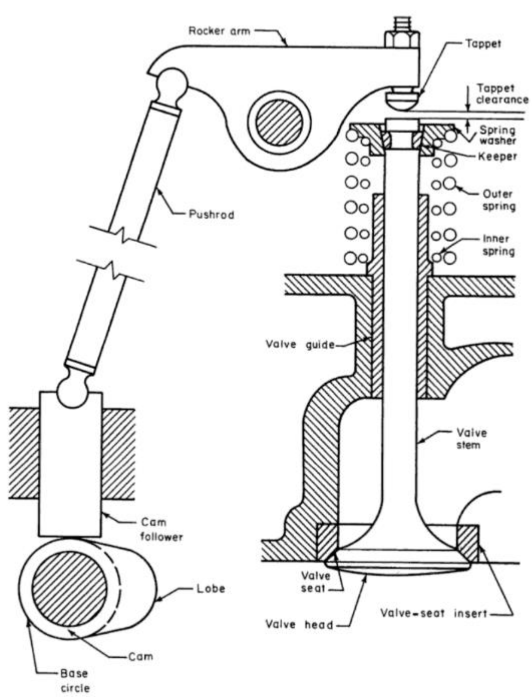


Fig.1. Valve Train Main Parts

After determine relations between cam performance parameter and acceleration jump. Other so important issue is valve spring force due to multifaceted influence. On cam profile, to be continuous contact, valve spring stiffness should be calculated carefully acc. to worse case of acceleration. Otherwise, valve spring may have wrong stiffness and may cause valve train working problem. The purpose of the valve spring is to close the valve in a controlled fashion. This requires maintaining constant contact among valve train components during valve movement. In the "valve closed" state, the spring force must be great enough to keep the valve from bouncing on the valve seat immediately after closing. In the "valve open" state, it is necessary to prevent "fly-over" i.e., the valve stem lifting off and breaking contact with the cam at maximum deceleration. Required spring force should not be so high level in order to be opened intake and exhaust valve without any deformation and breaking [2].

1. CAMSHAFT PROFILE DETERMINATION AND FORMULATION

According to different engine type and requirement, cam follower type, valve train system placement, cam profile type will be changed. In this paper, we will investigate and analyze one of the most used & known cam profile & follower type. Because of that system criteria which are valve train dwell, rise, return, velocity, acceleration affect directly engine efficiency and performance all parameters which cause these results should be observed by means of determination of cam profile.

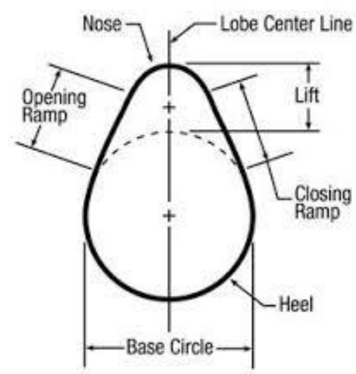


Fig.2. Common Cam Profile

There are three main parts in a cam profile: Heel, nose, Ramp (opening & closing Ramp). During design of cam profile, considered contact between cam follower and cam profile are ensured basically at two areas as base circle and nose.

When the flanks of the cam connecting the base circle and nose are of convex circular arcs, then the cam is known as circular arc cam. A symmetrical circular arc cam operating a flat-faced follower is shown in Fig.3. in which O and Q are the centers of cam and nose respectively. EF and GH are two circular flanks whose centers lie at P and P' respectively. The centers P and P' lie on lines EO and GO produced [1].

r1: Radius of the base circle r1=OE,
r2: Radius of nose,
R: Radius of circular flank R=PE,
2α: Total angle of action of cam 2α=Angle of EOG,
α: Semi-angle of action of cam α=Angle of EOK,
ϕ: Angle of action of cam on the circular flank,
Θ: From E to C angle of action of cam.

When the Flat Face of Follower has Contact on the Circular Flank: consider that the flat face of the cam follower has contact at E. When the cam turns through an angle ϕ (less than Θ) relative to the follower, the contact of the flat face of the follower will shift from E to C on the circular flank, such that flat face of the follower is perpendicular to PC. Since OB is perpendicular to BC, therefore OB is parallel to PC. From O, draw OD perpendicular to PC [1].

From the geometry of the figure, the displacement or lift of the follower (x) at any instant for contact on the circular flank is given by:

x =BA = BO – AO = CD – EO (1)
CD = PC – PD =PE – OP cosΘ
CD = OP + OE – OP cosΘ =OE + OP (1-cosΘ)
Substituting the value of CD in equation (1):
x = OE + OP (1-cosΘ) – EO =OP (1-cosΘ)
x = (PE – OE) (1 - cosΘ) = (R – r1) (1 - cosΘ) (2)

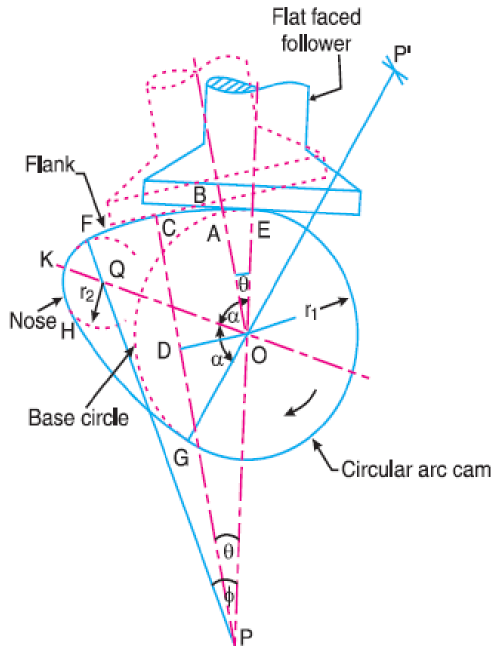


Fig.3. Contact of Cam Follower on the Circular Flank [1]

Cam Follower Velocity at Cam Opening & Closing Ramp: Differentiating equation (2) with respect to t, we have velocity of the follower,

$v = \frac{dx}{dt} = \frac{dx}{dQ}\frac{dQ}{dt}$

$[\frac{dQ}{dt} = \omega]$

$v = \frac{dx}{dQ}\,\omega$

v = (R- r1) sinΘ ω = (R- r1) sinΘ ω (3)

When Θ = 0, sinΘ = 0 and $v_{min} = 0$

As Θ increase, velocity of follower will increase.

When Θ = ϕ, velocity of follower will be maximum [v_{max}]

v_{max} = ω (R- r1) sinϕ

Cam Follower Acceleration at Cam Opening & Closing Ramp:

$a = \frac{dv}{dt} = \frac{dv}{dQ}\frac{dQ}{dt}$

$[\frac{dQ}{dt} = \omega]$

$a = \frac{dv}{dQ}\,\omega$

a = ω (R- r1) cosΘ ω = ω^2 (R- r1) cosΘ (4)

When Θ = 0, cosΘ = 1 and acceleration is maximum.

$a_{max} = \omega^2$ (R- r1)

When Θ = ϕ, acceleration is minimum.

$a_{min} = \omega^2$ (R- r1) cosϕ

When the Flat Face of Follower has Contact on the Nose: The flat face of the follower having contact on the nose at C is shown in Fig.4. The center of curvature of the nose lies at Q. In this case, the displacement or lift of the follower at any instant when the cam has turned through an angle Θ (greater than ϕ) is given by:

x = AB = BO – OA = CD – OA (5)
CD = CQ + QD = CQ + OQ cos (α - Θ)
Substituting the value of CD in equation (5):
The displacement or lift of the follower when the contact is at the K of the nose, when α - Θ = 0 and cos (α - Θ) = 1
x = CQ + OQ cos (α - Θ) – OA = r2 – r1 + OQ cos (α - Θ) (6)

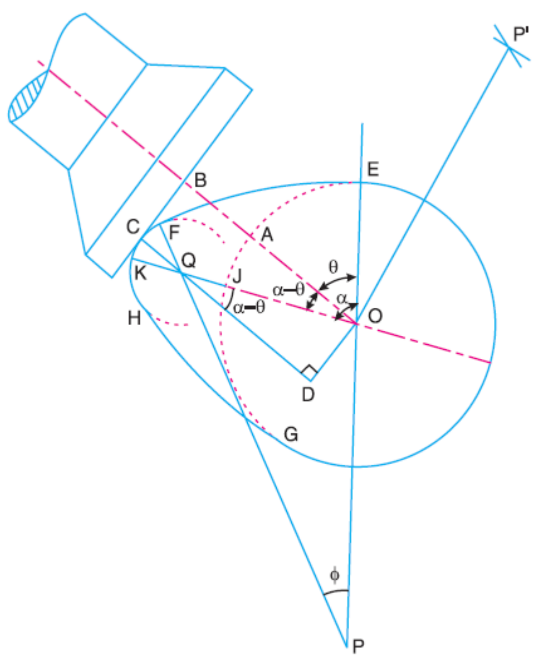


Fig.4. Contact of Cam Follower on the Nose [1]

Cam Follower Velocity at Cam Nose

$v = \frac{dx}{dt} = \frac{dx}{dQ}\frac{dQ}{dt}$

$[\frac{dQ}{dt} = \omega]$

$v = \frac{dx}{dQ}\omega$

$v = OQ \sin(\alpha - \Theta)\,\omega$ (7)

When $\alpha - \Theta = 0$, at K of the nose, velocity is zero and minimum. When $(\alpha - \Theta)$ is maximum, the velocity will be maximum. When the follower changes contact from circular flank to circular nose at point F, when $\alpha - \Theta = \phi$.

Cam Follower Acceleration at Cam Nose

$a = \frac{dv}{dt} = \frac{dv}{dQ}\frac{dQ}{dt}$

$[\frac{dQ}{dt} = \omega]$

$a = \frac{dv}{dQ}\omega$

$a = -\omega^2 OQ \cos(\alpha - \Theta)$ (8)

When $\alpha - \Theta = 0$, $\cos(\alpha - \Theta) = 1$, at the K of the nose and acceleration is maximum. The negative sign in the above expression shows that there is retardation when the follower is in contact with the nose of cam.

Maximum retardation; $a_{max} = \omega^2 OQ$

The retardation is minimum when $\alpha - \Theta$ is maximum. This happens when the follower changes contact from circular flank to circular nose at point F. when $\alpha = \phi$

Thanks to these above formulization, it will be possible to plot graphs of follower lift, velocity, acceleration. So at acceleration graph, changes from maximum value to minimum value will be observed and analyze on how to be reduced with parameter change will be carried out.

Specifications of our engine which we use for this paper as mentioned below:

i = 1 (Single Cylinder)	$n_{nominal} = 2250\ rpm$
4 Stroke	B = 108 mm (Cylinder Bore)
Naturally Aspirated	H = 127 mm (Stroke Height)
Water Cool Engine	ε = 1:18 (Compression Ratio)

$P_c = 0,1\ MPa$ (combustion chamberAir Intake Pressure)

Intake - Exhaust Timing Value:

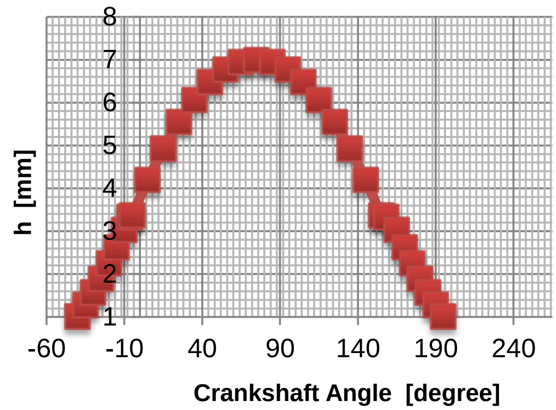
Timing of Intake Valve open	α1 = 60 KMA
Timing of Intake Valve close	α2 = 80 KMA
Timing of Exhaust Valve open	α3 = 80 KMA
Timing of Exhaust Valve close	α4 = 60 KMA

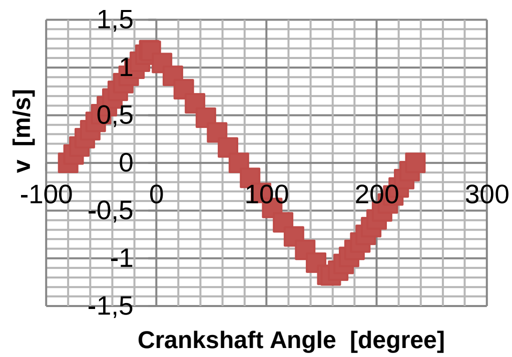
Valve Maximum Lift: h "JK"	7
Rocker arm: l_1(Cam Follower Side)	45,65
Rocker arm: l_2 (Valve Side)	69,8
Rocker arm ratio: l_1/ l_2	0,654011
Maximum lift at cam follower side: h_k	4,57808
Gap between valve and rocker arm: s "Experimental@800celcius"	0,55
Gap between rocker arm and push-rod: s_k	0,359706
Cam shaft radius(acc.to r_1): r_c	22,14029
Base circle radius: r_1	22,5
Nose radius: r_2	14
r_1 -r_2	8,5
OQ= JK+r_1 -r_2	15,5
OP	16,76
PQ=PF-FQ=PE-FQ=OP+OE-FQ PQ=OP+r_1 -r_2	25,26
Radius of the circular flank: R= OP+r_1	39,26
sin ϕ ==OQ*sinβ/PQ	0,60
ϕ_1 = ϕ max.	36,72653
ϕ_2 = α-ϕ_1	40,27347
Timing of Exhaust Valve open [Crank Angle]	80
Timing of Exhaust Valve close [Crank Angle]	60
Crankshaft Rated Speed: n (rpm)	2250
Crankshaft Angular Speed: ω (Radian / sec.)	235,62
Valve Lift Angle: α [Crank Angle]	77
β: (180- α)	103
Camshaft Rated Speed: ncam (rpm)	1125
Camshaft Angular Speed: ωcam (Radian / sec.)	117,8097

Table 1. Calculated Cam Profile Values

CAM PARAMETER AND PERFORMANCE GRAPHS

Acc. to calculated cam profile values table, we can create performance graph of cam follower.





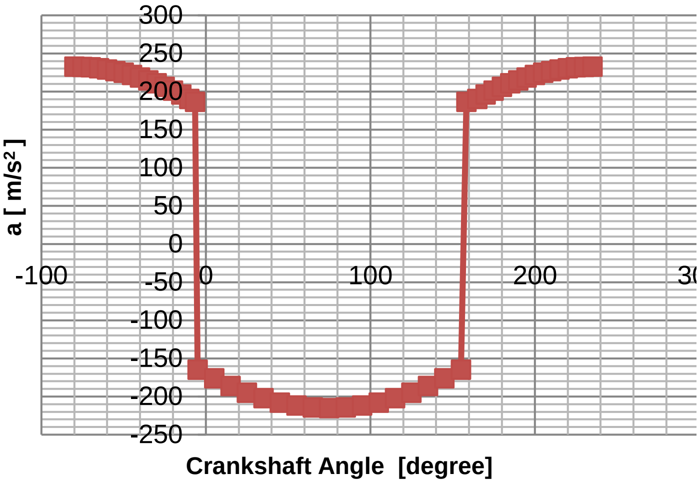


Fig.5. Cam Performance Graphs [h, v, a]

After plotting performance graphs, it is important to determine relations between parameters and performance. As we mentioned, because of that acceleration jump causes the worst case in terms of action of cam profile. By means of Matlab, cam profile parameters will be analyzed as follows;

Basic reason of acceleration jump is sudden change from acceleration with positive sign to deceleration with negative sign. This acceleration jump also may cause loose contact between cam follower and cam profile and so it may do damage on system equipment. When designing the valve lift curve the designer should ensure that the jerk and quirk are minimized within the constraints imposed by performance requirements [3].

Graphs shown below plots with a difference between maximum and minimum acceleration value and change of main parameter by using Matlab.

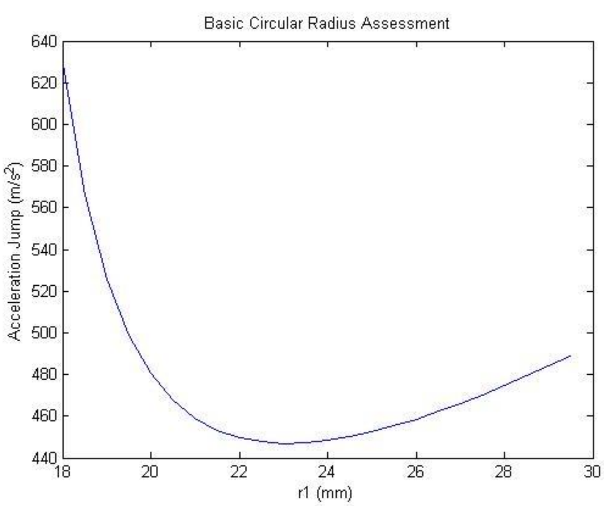


Fig.6. Acceleration jump-BaseCircle Radius

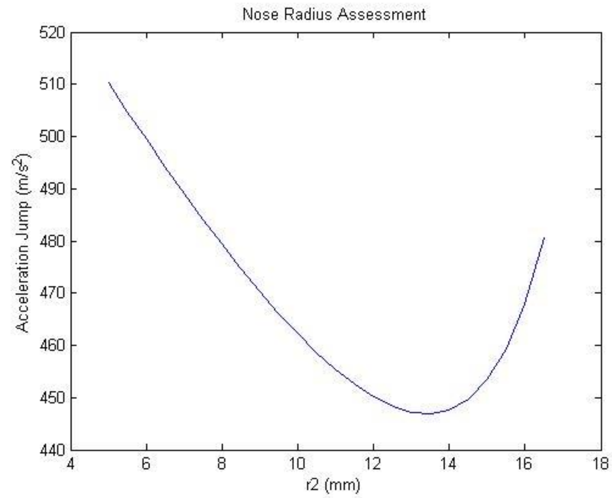


Fig.7. Acceleration jump-Nose Radius

Acc.to shown graphs when base circle radius & nose radius increases up to a certain value, acceleration jump decreases but after a certain value of base circle & nose radius, acceleration jump again increases. That's why, during design process, it should be found out these min. values.

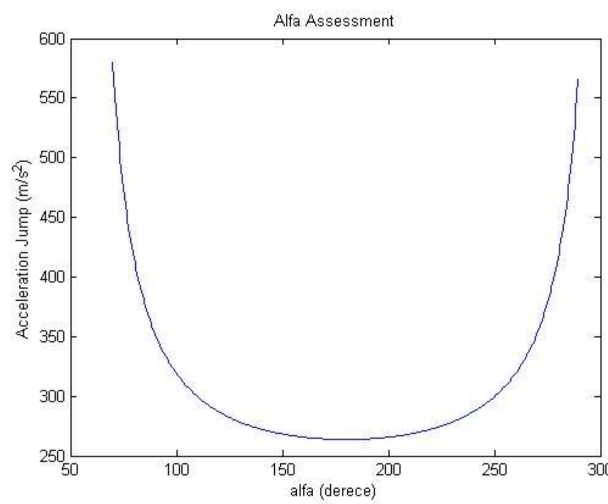


Fig.8. Acceleration jump-Valve Lift Angle

Acc.to graph shown above, minimum acceleration occurs at 180 degree of cam shaft angle. When it comes closer to 180 degrees, acceleration jump decreases. When it goes far from 180 degrees, acceleration jump increases. Because of valve lift angle is affected by valve intake & exhaust timing open-close, we may not select valve lift angle which corresponding min. acceleration value. Valve lift angle is suggested to determine as follows:

$2\varphi_{Intake\&Exhaust} = (180 + \alpha 1 + \alpha 2)/2$

$\alpha 1$ & $\alpha 2$ is timing of valve open-close

Valve lift angle $=(2\varphi_{\text{Intake\&Exhaust}})$ / 2

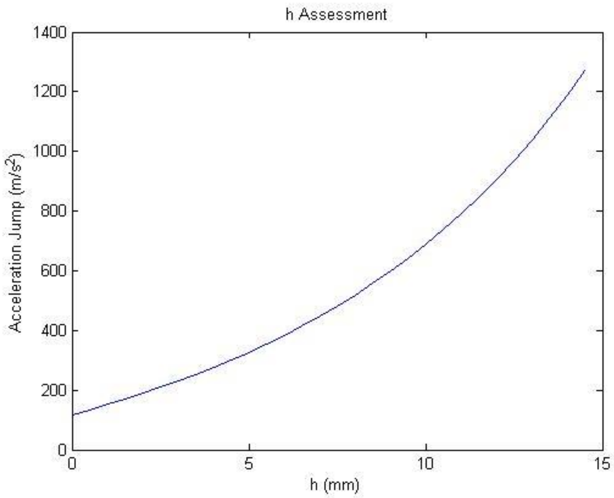


Fig.9. Acceleration jump-Max. Valve Lift

Acc.to graph of acceleration jump-max. valve lift, when max. valve lift increases, acceleration jump increases. Designer should make a decision to catch optimal result. Because of that increase of max. valve lift means that increase of volumetric efficiency. But this increase cannot be endless. It is limited by engine out size and acceleration jump.

DETECTION OF VALVE SPRING STIFFNESS AND FORCE

The most important forcing factor for valve spring is not loose contact between cam and cam follower. As it is understood from acceleration graph which was changed suddenly form positive sign to negative sign, around these areas are critical and dangerous for system. It should be focused on to be catch actual valve stiffness.

When acceleration sign is positive, cam follower speeds up and it does not happen any problem in terms of contact. But when acceleration turns from positive sign to negative sign, so this is so high speed and sudden, at this time cam follower tends to speed up. Otherwise, cam tries to deceleration. This very moment, it is seen conflict at behavior of cam follower affected by inertia moment.

This loose of contact is instant and short time then cam follower velocity slow down and ensured contact between cam and cam follower. Even though this loose contact is short time, causes to be high level of inertia of cam follower, also make hard collision between them. That's why, level of noise increase, it can be seen breaking and damage on system. Interruptedly motion of valve may be caused not to take enough weather at intake stroke so combustion efficiency decreases. Beside this, at exhaust stroke, it may be problem during evacuation of exhaust gas, so fall of efficiency occurs.

If needed, to set out of a dynamic model and see system kinematic behavior, valve spring stiffness is determinant due to effect of natural frequency, effect on cam contact force & torque as dynamic and static, contact angle. Basic question is how it should be calculated:

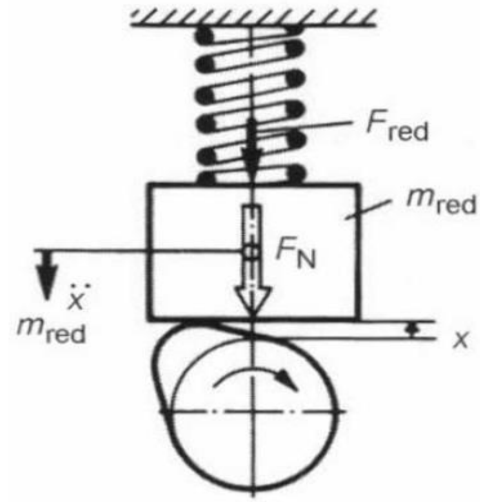


Fig.10. Reduction of Valve Spring Force on Cam Follower

The most critical situation at determination of valve stiffness is change of acceleration from positive to negative. Designer should consider this lift value corresponding to change of acceleration, while taking account into needed force which supply with inertia moment at critical point.

In order to be calculated valve stiffness, first of all, it should be applied mass reduction for system parts as mentioned below:

$$M_v = m_v + \frac{m_s}{3} + \frac{m_b}{2} + [(m_f + m_{pr})(\frac{l_f}{l_v})^2 + m_r'] / 2 \qquad (9)$$

$$m_r' \cong \frac{m_r(l_v+l_f)^2}{12 l_v^2} \qquad (10)$$

M_v: Reduction Mass on Valve
m_v: Valve Mass (İncluded Valve, Spring Seat and Spring Retainer)
m_s: Spring Mass
m_b: Valve Bridge Mass (It is ignored during preliminary calculation)
m_f: Cam Follower Mass
m_{pr}: Push-Rod Mass
m_r': Reduction Mass of Rocker Arm
m_r: Rocker Arm Mass
l_v: Rocker Arm – Valve Side
l_f: Rocker Arm – Cam Follower Side
For exhaust valve spring;
m_v: 0,127kg
m_s: 0,047kg (For preliminary calculation, if it is unknown accept 0,01Kg.)
m_f: 0,151kg
m_{pr}: 0,155kg
m_r: 0,244kg
l_v: 45,65mm
l_f: 69,8mm

$$m_r' \cong \frac{0{,}244(45{,}65 + 69{,}8)^2}{12(45{,}65)^2}$$

m_r': 0,130051kg

$M_v = 0{,}127 + \frac{0{,}047}{3} + [(0{,}151 + 0{,}155)(\frac{69{,}8}{45{,}65})^2 +$
$0{,}130051] / 2$
M_v: 0,565393kg

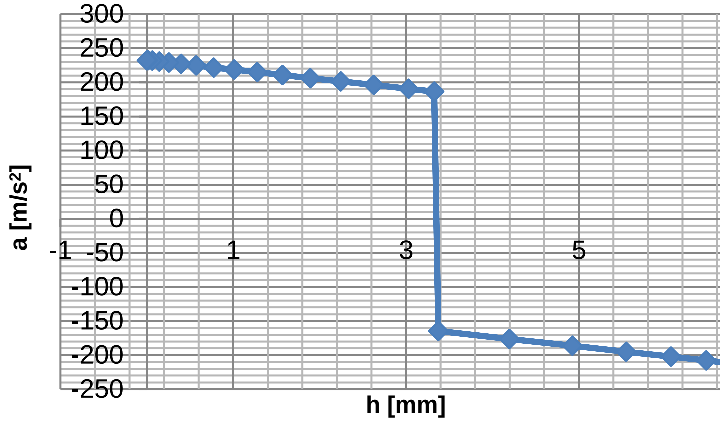


Fig.11. Change of acceleration suddenly-Valve Lift

Acc. to above graphs, valve lift value corresponding change of acceleration from positive sign to negative sign suddenly is 3,326mm. Critical acceleration is 186, 3907581 m/s^2. Considering these found values;

$$F_s = s.M_v.a_c.\frac{l_v}{l_f} \quad (11)$$

F_s: Valve Spring Force [N]
a_c: Critical acceleration jump [m/s^2]
s: Safety Factor

$F_s = 1{,}5x0{,}565393x186{,}3907581x\frac{45{,}65}{69{,}8}$
$F_s \cong 105N$
$F_s = k_s.x_s$,
$k_s = F_s / x_s$
k_s: Valve spring stiffness [N/mm]
x_s: Valve spring total displacement [mm]

Valve spring total displacement means total of pre-deflection under preload and the displacement value corresponding acceleration jump point.

Pre-deflection affects directly valve spring stiffness. While changing of pre-deflection of valve spring under preload, stiffness can be adjusted.

Pre-deflection of valve spring accepts as 10mm.

x_s= 10 + 3,326 = 13,326mm
k_s = 105 / 13,326
$k_s \cong 7{,}8793\ [\frac{N}{mm}]$

While this stiffness base on, main valve spring size can be determined:

τ: Torsional Stress on Valve Spring Wire $[\frac{N}{mm^2}]$
F_s: Valve Spring Force [N]
D_s: Valve Spring Main Diameter [mm]
d_w: Valve Spring Wire Diameter [mm]
k: Correction Factor for Stress

$$\tau = \frac{8.D_s.F_s}{\pi.d_w^3}$$

τ_{em}: 450MPa Max. Safety Stress (50CrV4 for Spring Steel)
D_s: 23mm (Considering valve stems dimension for pre-calculate)

$d_w = \sqrt[3]{\frac{8.D_s.F_s}{\pi.\tau_{em}}}$

$d_w = \sqrt[3]{\frac{8 \text{ x } 23 \text{ x } 105}{\pi \text{ x } 450}} = 2{,}39082\text{mm}$

$d_w \cong 2{,}4\text{mm}$

$k = \frac{4c-1}{4c-4} + \frac{0{,}615}{c}$, (Correction factor for stress, especially if it is used greater than 10.000.000 cycle.) By Bergstrasser.

$c = \frac{D_s}{d_w}$

$c = \frac{23}{2{,}4} = 9{,}583$

$k = \frac{4x9{,}583-1}{4x9{,}583-4} + \frac{0{,}615}{9{,}583} = 1{,}1515$

$\tau_{max} = k.\frac{8.D_s.F_s}{\pi.d_w^3} = 1{,}1515 \text{ x } \frac{8 \text{ x } 23 \text{ x } 105}{\pi.2{,}4^3} \cong 512\text{Mpa}$

Due to $\tau_{max} \geq \tau_{safe}$, d_w should be increased.
If considered d_w = 3 mm;

$c = \frac{23}{3} = 7{,}666$

$k = \frac{4x7{,}666-1}{4x7{,}666-4} + \frac{0{,}615}{7{,}666} = 1{,}1927$

$\tau_{max} = k.\frac{8.D_s.F_s}{\pi.d_w^3} = 1{,}1927 \text{ x } \frac{8 \text{ x } 23 \text{ x } 105}{\pi.3^3} \cong 272\text{Mpa}$

Due to $\tau_{max} \cong$ 272Mpa $\leq \tau_{safe}$ = 450Mpa, it is suitable.

$s = \frac{\tau_{safe}}{\tau_{max}}$ safety factor ≅1,65

i: Number of Wire Turn
f_{max} : Valve Spring Max. Deflection [mm]
f_{max} : Total of max. Valve Lift and Pre-deflection:7+10=17mm
G: Valve Spring Shear Modules [G=83000 Mpa]

$i = \frac{d_w.f_{max}.G}{\pi.D_s^2.\tau}$

$i = \frac{3 \text{ x } 17 \text{ x } 83000}{\pi \text{ x } 23^2.272} = 9{,}3690$ (accepted 10)

H: Valve Spring Free Height
Δ: Gap Among Pressed Spring Wire (0,2...0,55)mm.
i^n: Number of Inactive Spring Wire Turn (1&2)
$H = (d_w + \Delta)\ i + f_{max} + i^n\ d_w$
$H = (3 + 0{,}5)\ x\ 10 + 17 + 2 \text{ x } 3 = 58\text{mm}$

After calculation of main valve spring size, it should be checked natural frequency of valve spring in order not to be any conflict. If there is, valve spring size should be changed by designer otherwise it may be come across resonance.

$$w_{spring} = \frac{21{,}7x10^6xd_w}{i.D_s^2} \quad (12)$$

$w_{spring} = \frac{21{,}7\ x\ 10^6\ x\ 3}{10.23^2} = 56710{,}7750$ [1/minute]

w_{spring} = 945,1795 (Radian/sec.) & w_{cam} = 117,8097245 (Radian / sec.)

Due to [$w_{spring} \neq w_{cam}$], it can be seen that it will not be problem in terms of resonance.

STATIC CALCULATION OF CAMSHAFT DIAMETER

Therein before, it was mentioned relation between base circle radius and acceleration jump. Acc. to related graph, after that 23mm of base circle radius, when it keeps on increasing, acceleration jump increases, too. So designers can indicate its optimal result is around 22mm. Acc. to only these graphs results, it should not be decided that it should not to be increased. Because it can be necessary to increase base circle radius although acceleration jump increases due to camshaft strength requirement. Base circle radius is so strongly related with camshaft diameter. Before determine of base circle radius, it should be suggested to calculate of camshaft static analysis as shown below:

$F_{N\ Intake\&\ Exhaust}: M_v.\ a_{C_{maks}} + F_s.\frac{l_f}{l_v}$ (13)

$F_{N\ Intake}: 0{,}54489\ x\ 184{,}52 + 53\ \mathrm{x}\ \frac{53{,}7}{35{,}5} \cong 180{,}7N$

$F_{N\ Exhaust}: 0{,}565393\ x\ 232{,}553 + 105\ \mathrm{x}\ \frac{69{,}8}{45{,}65} \cong 292{,}03N$

$F_{N\ Intake\&\ Exhaust}$: Contact Force on Cam Profile.

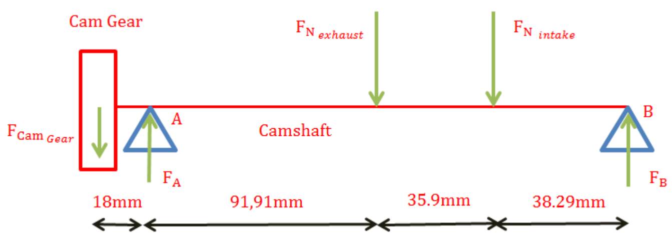


Fig.12. Forces on Camshaft

Cam gear mass is 1,812kg and when we have added 0,247kg of related parts mass. Total cam gear mass is 2,059kg.

$F_{Cam\ Gear} = G_{\ Cam\ Gear} = \mathrm{m\ x\ g} = 2{,}059\mathrm{kg\ x}\ 9{,}81\ \frac{\mathrm{m}}{\mathrm{s}^2} = 20{,}198\mathrm{N}$

Camshaft mass is measured as 1,813kg. For pre-calculate, due to non-homogeny cam structure and it is not so affected for static analysis, camshaft mass is ignored.

$\sum F_Y=0$; because vertical force is balanced and zero.

$F_{N\ exhaust} + F_{N\ intake} + F_{Cam\ Gear} = F_A + F_B$

F_A and F_B : Reaction force [N]

$F_A + F_B = 292{,}03\mathrm{N} + 180{,}7\mathrm{N} + 20{,}198\mathrm{N} = 492{,}928\mathrm{N}$

$\sum$ M=0; because moment is balanced and zero.

$\sum M_B=0$, Moment acc. to B point.

$F_A \ \mathrm{x}\ 166{,}1 = F_{N\ exhaust}\ \mathrm{x}\ 74{,}19 + F_{N\ intake}\ \mathrm{x}\ 38{,}29$
$+\ F_{Cam\ Gear}\ \mathrm{x}\ 184{,}1$

$F_A = (292{,}03\ \mathrm{x}\ 74{,}19 + 180{,}7\ \mathrm{x}\ 38{,}29 + 20{,}198\ \mathrm{x}\ 184{,}1)\ /\ 166{,}1$

$F_A = 194{,}480\mathrm{N}$

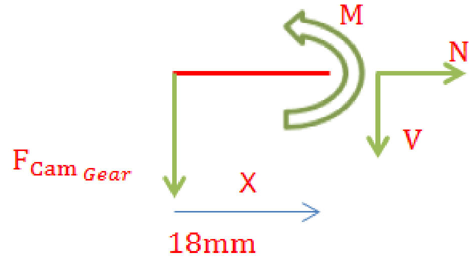
$F_A + F_B = 492{,}928\mathrm{N}$

$F_B = 492{,}928\mathrm{N} - F_A = 492{,}928\mathrm{N} - 194{,}480\mathrm{N}$

$F_B = 298{,}448\mathrm{N}$

Acc. to above results, Section Force Diagrams:

I.Area



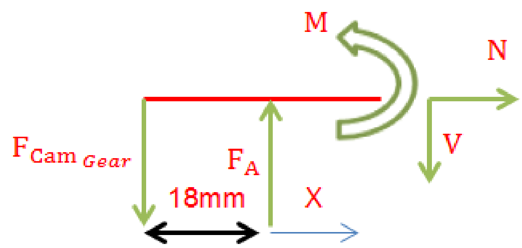
$V = -\ F_{Cam\ Gear} = -\ 20{,}198\mathrm{N}$ Shear Force

$N = 0$

$M = -\ F_{Cam\ Gear}\ \mathrm{x} = -20{,}198\mathrm{N}\ 18\mathrm{mm} = -363{,}564\mathrm{Nmm}$

$0 \le x \le 18$

II.Area



$V = F_A - F_{Cam\ Gear} = 194{,}480\mathrm{N} - 20{,}198\mathrm{N} = 174{,}282\mathrm{N}$

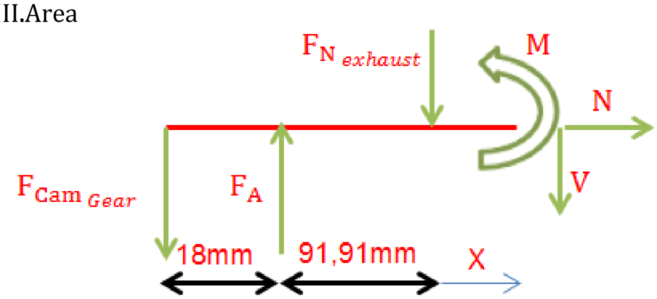
$N = 0$

$M = -F_{Cam\ Gear}\ (\mathrm{x} + 18) + F_A\ \mathrm{x} = -20{,}198\mathrm{N}\ (18\mathrm{mm} + 91{,}91\mathrm{mm}) + 194{,}480\mathrm{N}\ 91{,}91\mathrm{mm}$

$M = 15654{,}694\mathrm{Nmm}$

$0 \le x \le 91{,}91$

III.Area



$V = F_A - F_{Cam\ Gear} - F_{N\ egzoz} = 194{,}480\mathrm{N} - 20{,}198\mathrm{N} - 292{,}03\mathrm{N} = -117{,}748\mathrm{N}$

$N = 0$

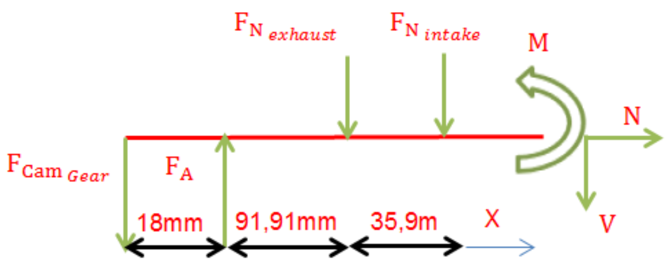
$M = -\ F_{Cam\ Gear}\ (\mathrm{x} + 109{,}91) + F_A\ (\mathrm{x} + 91{,}91) - F_{N\ exhaust}\ \mathrm{x}$

$M = -20{,}198\mathrm{N}\ (35{,}9 + 109{,}91) + 194{,}480\mathrm{N}(35{,}9 + 91{,}91) - 292{,}03\mathrm{N}\ 35{,}9\mathrm{mm}$

$M = 11427{,}541\mathrm{Nmm}$

$0 \le x \le 35{,}9$

IV.Area



$V = F_A - F_{Cam\ Gear} - F_{N\ exhaust} - F_{N\ intake} = 194{,}480N - 20{,}198N - 292{,}03N - 180{,}7N$
$V = -298{,}448N$
$N = 0$
$M = - F_{Cam\ Gear}\ (x + 145{,}81) + F_A\ (x + 127{,}81)$
$- F_{N\ exhaust}\ (x + 35{,}9) - F_{N\ intake}\ x$
$M = - 20{,}198N\ (38{,}29 + 145{,}81) + 194{,}480N\ (38{,}29 + 127{,}81) - 292{,}03N\ (38{,}29 + 35{,}9) - 180{,}7N\ 38{,}29 = 0$
$0 \le x \le 38{,}29$

Shear Fore: V [N]
Torque: M [Nm]
Normal Force: N [N]

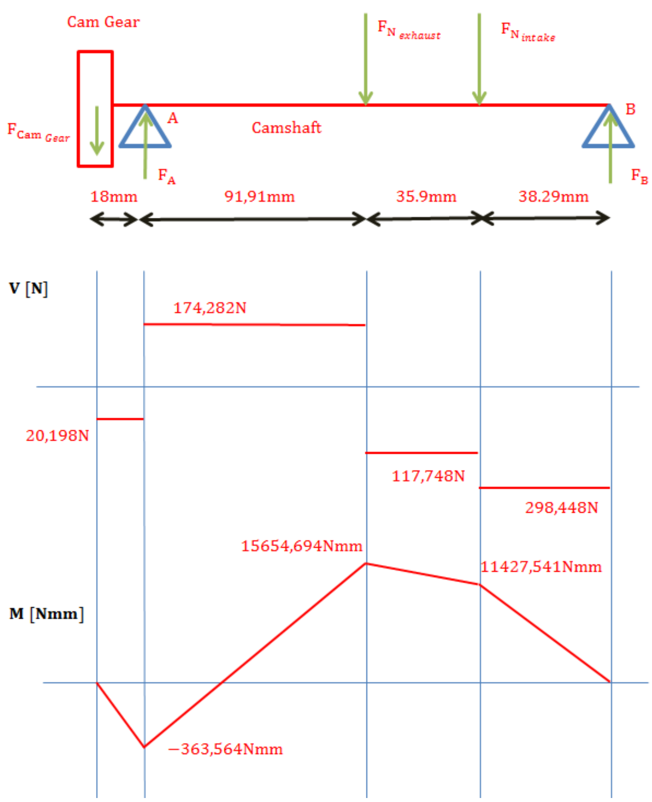


Fig.13. Section Force & Moment Diagram For CamShaft

$\widetilde{\sigma_e}$: Dynamic Bending Stress
$\overline{\sigma_e}$: Static Bending Stress
$\widetilde{\sigma_v}$: Dynamic Equivalent Stress
$\overline{\sigma_v}$: Static Equivalent Stress
$\overline{\tau_b}$: Static Torsional Stress
$\widetilde{\tau_b}$: Dynamic Torsional Stress
M_e: Maximum Static Bending Stress
M_b: Maximum Static Torsional Stress
W_e: Section Modulus for Bending
W_b: Section Modulus for Torsion
$\widetilde{\sigma_{ç,b}}$: Dynamic Tensile & Compressive Stress
$\overline{\sigma_{ç,b}}$: Static Tensile & Compressive Stress

For camshaft material is GG30:
$\sigma_{ç,b}$: Ultimate Tensile Strength $(400\ \frac{N}{mm^2})$
σ_e: Ultimate Bending Strength $(480\ \frac{N}{mm^2})$
$\sigma_{eq_{safe}}$: $90\ \frac{N}{mm^2}$ (Max. Equivalent Stress Thanks to Main Strength Diagram)
For Static Equivalent Stress:
$\overline{\sigma_v} = \sqrt{(\overline{\sigma_{ç,b}} + \overline{\sigma_e})^2 + 3\overline{\tau_b}^2}$ at static equivalent stress, there is no tensile & compressive stress due to there is no axial force. Also characteristic of bending stress for system is dynamic not static.
$$\overline{\sigma_v} = \sqrt{(\overline{\sigma_{ç,b}} + \overline{\sigma_e})^2 + 3\overline{\tau_b}^2} = \sqrt{3}\ \overline{\tau_b}$$
For Dynamic Equivalent Stress:
$\widetilde{\sigma_v} = \sqrt{(\widetilde{\sigma_{ç,b}} + \widetilde{\sigma_e})^2 + 3\widetilde{\tau_b}^2}$ at dynamic equivalent stress, there is no tensile & compressive stress due to there is no axial force. Also characteristic of torsional stress for system is static not dynamic
$$\widetilde{\sigma_v} = \sqrt{(\widetilde{\sigma_{ç,b}} + \widetilde{\sigma_e})^2 + 3\widetilde{\tau_b}^2} = \widetilde{\sigma_e}$$
$$\widetilde{\sigma_e} = \frac{M_e}{W_e}$$
$M_e = 15654{,}694$ (Max. bending stress acc.to calculated Section Diagram)
$$W_e = \frac{\pi d^3}{32}$$
$$\widetilde{\sigma_e} = \frac{15654{,}694\text{Nmm}}{\frac{\pi d^3}{32}}$$
$$\overline{\tau_b} = \frac{M_b}{W_b}$$
$M_b = 9550\ \frac{P}{n}$ calculated by P = 16 KW (Power) and n = 1125 rpm (Camshaft rated speed).
$$M_b = 9550\ \frac{16}{1125} = 135{,}822\text{Nm} = 135822\text{Nmm}$$
$$W_b = \frac{\pi d^3}{16}$$
$$\overline{\tau_b} = \frac{135822Nmm}{\frac{\pi d^3}{16}}$$
$$\widetilde{\sigma_v} + \overline{\sigma_v} \le \sigma_{eq_{safe}} \qquad (14)$$
$$\widetilde{\sigma_e} + \sqrt{3}\ \overline{\tau_b} \le \sigma_{eq_{safe}}$$
$$\frac{15654{,}694\text{Nmm}}{\frac{\pi d^3}{32}} + \sqrt{3}\ \frac{135822Nmm}{\frac{\pi d^3}{16}} \le 90\ \frac{N}{mm^2}$$

Acc. to safety factor [s] is 1, camshaft diameter $d \geq 24{,}708$ mm
Acc. to safety factor [s] is 1.5, camshaft diameter $d \geq 28{,}287$ mm
Acc. to safety factor [s] is 2, camshaft diameter $d \geq 31{,}134$ mm
Acc. to safety factor [s] is 3, camshaft diameter $d \geq 35{,}640$ mm
While designers change base circle diameter to get optimal result in terms of less acceleration jump, should consider camshaft diameter as shown above. It is not possible to choose less of base circle diameter than camshaft diameter. It means that base circle diameter depends on camshaft diameter directly.

THEORITICAL CONTACT FORCE CALCULATION

As it can be seen, to be calculated camshaft diameter should be known contact force. At the same time contact force is related with cam torque and affects systems behavior. That's why, it is important to see how it changes acc. to crankshaft angle.

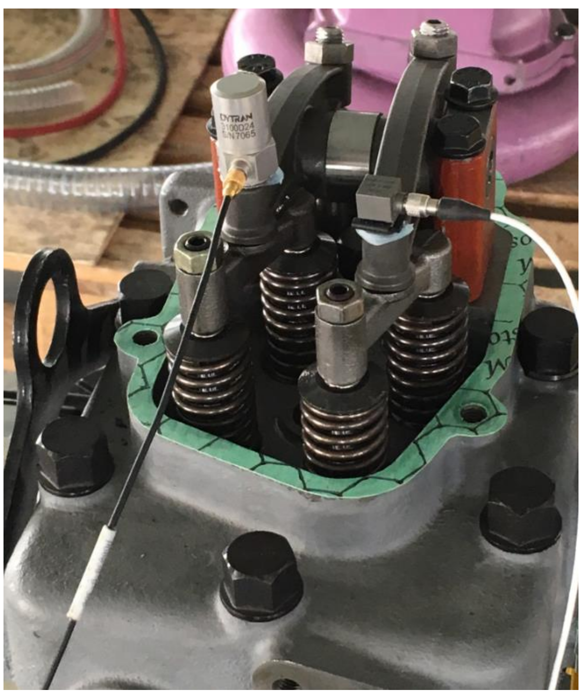


Fig.14. Actual Natural Frequency Test of Mentioned Engine

Acc.to actual test results applied on valve spring (Data coming from actual engine):
$L_0 = 64\text{mm} \pm 0{,}5mm$ (Valve Spring Free Height)
$D = 22{,}5\text{mm}$ (Valve Spring Main Diameter [mm])
$d = 3{,}4\text{mm}$ (Valve Spring Wire Diameter [mm])
$P_1 = 21 \pm 1kg$ (First Valve Spring Load)
$P_2 = 43{,}55 \pm 2kg$ (Second Valve Spring Load)
$L_1 = 50\text{mm}$ (Spring Height under First Load)
$L_2 = 35\text{mm}$ (Spring Height under Second Load)
When P_1 applied on valve spring:
$k_1 = k_2 = \frac{F}{x} = \frac{P_1}{L_0 - L_1} = \frac{21kg}{(64-50)mm} = 1{,}5$ kg/mm
When P_2 applied on valve spring:
$k_1 = k_2 = \frac{F}{x} = \frac{2}{L_0 - L_2} = \frac{43{,}55kg}{(64-35)mm} \cong 1{,}5$ kg/mm
So it is found $k_1 = k_2 = 14{,}715$N/mm acc.to actual test results.
k_1 & k_2 : Valve Spring Stiffness
x: Pre-deflection of valve spring
F_p: Preload Spring Force [N]
$F_p = k\,x = 14{,}715\text{N/mm}\ 10\text{mm} = 147{,}15\text{N}$
F_c : Contact force [N]
k_h : Stiffness between cam profile and cam follower due to contact [N/m]
$y(\theta)$: Contact Point displacement on cam profile [mm]
y_1 : Cam vertical displacement [mm]
y_2 : Cam follower vertical displacement [mm]
α : Pressure Angle [°]

$$F_c = k_h\ (y(\theta) + y_1 + y_2)\cos\alpha + F_p \cos\alpha \qquad (15)$$

$\dot{Q}_c$: Camshaft angle speed [radian/sec.]
R_a : $R_b + R_1$ [mm]
R_b : Cam base circle radius [mm]
R_1 : Cam follower roller radius [mm]
$\dot{y_2}$ = Cam follower Speed [mm/sec.]
$\dot{Q}_c$ @1125rpm = 1125 rated speed = (1125/60)*2π [1/sec.] =117, 8[1/sec.]
$\dot{Q}_c$ @750rpm = 750 rated speed = (750/60)*2π [1/sec] =78,539 [1/sec]
$R_b = 22{,}5$mm
$R_1 = 10$mm
$R_a = R_b + R_1 = 22{,}5 + 10 = 32{,}5$mm
k_h value in some studies can be considered as changeable but for this paper, it is accepted is 1,9x10. [4], [5], [6], [7]

$$\alpha = \arctan\left(\frac{\dot{y_2}}{\dot{Q}_c\ (R_a + y(\theta))}\right) \qquad (16)$$

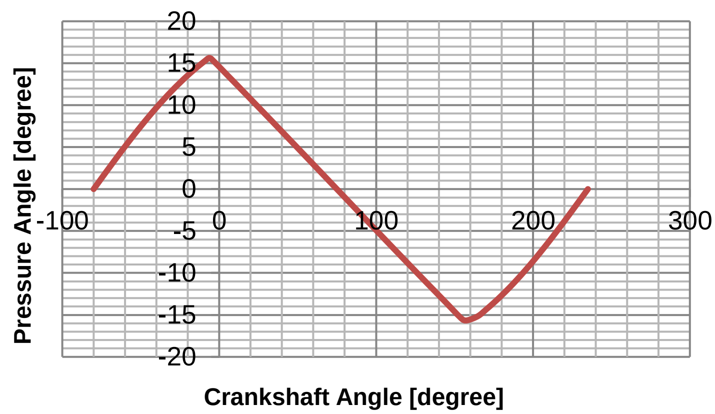


Fig.15. Relation Between Pressure Angle and Crank shaft Angle

Acc. to above graphs, pressure angle is approximately between +15 degree and -15 degree.
Contact forces are calculated for @750rpm & @1125rpm and results are as follows:

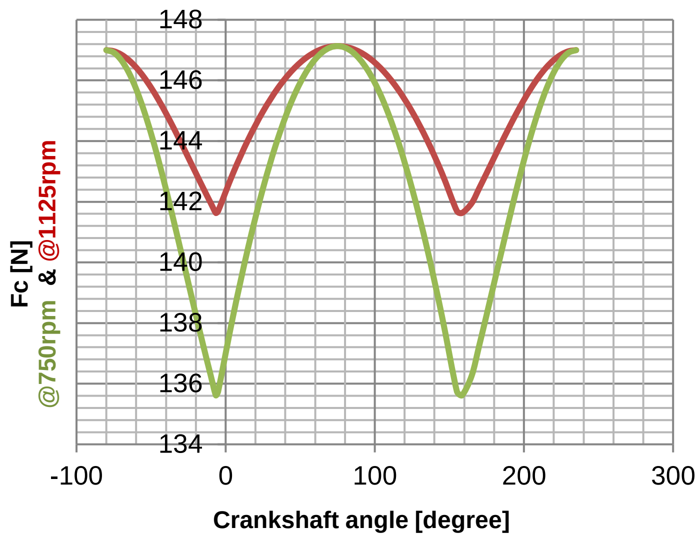


Fig.16. Contact Force @750rpm & @1125rpm

CONCLUSION

In this paper, all design criteria and parameters which affect cam performance and behavior of cam are investigated by Matlab, analytical, section force & moment analysis etc. Especially at pre-calculation and before actual test, these analysis and parameters should be determined after that tried to improve the system.

Optimal values of these pre-designs for instance; base circle radius, nose radius, valve max. lift, valve angle lift, valve spring stiffness, contact force calculations prepare to get the best results before actual test.

To be known behavior of cam and see strengths and weaknesses side of system is so important to go towards problematic issue. So that, optimal cam profile, cam also valve spring can be provided and valve train system will be able to run trouble-free.

NOMENCLATURE

Matlab: It means [Matrix Laboratory], and a program which supply operation with matrix solve.

Rpm: Revolutions per Minute.

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